Exercice 84 page 255

- $z_1 z_2 = e^{i\frac{\pi}{3}} \times 3e^{-i\frac{\pi}{4}}$

$= 3e^{i\frac{\pi}{3} - i\frac{\pi}{4}}$

$= 3e^{i\left(\frac{\pi \times 4}{3\times 4} - \frac{\pi \times 3}{4 \times 3}\right)}$

$= 3e^{i\frac{4\pi - 3\pi}{12}}$

$\boxed{z_1 z_2 = 3e^{i\frac{\pi}{12}}}$

- $\frac{z_1}{z_2} = \frac{e^{i\frac{\pi}{3}}}{3e^{-i\frac{\pi}{4}}}$

$= \frac{1}{3} e^{i\frac{\pi}{3} - \left(-i\frac{\pi}{4}\right)}$

$= \frac{1}{3} e^{i\left(\frac{\pi}{3} + \frac{\pi}{4}\right)} = \frac{1}{3} e^{i\left(\frac{4\pi + 3\pi}{12}\right)}$

$\boxed{\frac{z_1}{z_2} = \frac{1}{3} e^{i\frac{7\pi}{12}}}$

- $z_1^3 = \left(e^{i\frac{\pi}{3}}\right)^3$

$z_1^3 = e^{i\left(\frac{\pi}{3}\times 3\right)}$

$\boxed{z_1^3 = e^{i\pi}}$

- $z_1 z_2 z_3 = 3e^{i\frac{\pi}{12}} \sqrt{2} e^{i\frac{2\pi}{3}}$

$= 3\sqrt{2}\, e^{\left(\frac{\pi}{12} + \frac{2\pi}{3}\right)}$

$= 3\sqrt{2} e^{i\left(\frac{\pi}{12} + \frac{8\pi}{12}\right)}$

$\boxed{z_1 z_2 z_3 = 3\sqrt{2} e^{i\frac{3\pi}{4}}}$

- $z_3^4 = \left(\sqrt{2} e^{i\frac{2\pi}{3}}\right)^4$

$= \left(\sqrt{2}\right)^4 \left(e^{i\frac{2\pi}{3}}\right)^4$

$= \sqrt{2} \times \sqrt{2} \times \sqrt{2} \times \sqrt{2}\, e^{i\frac{2\pi}{3}\times 4}$

$\boxed{z_3^4 = 4e^{i\frac{8\pi}{3}}}$

- $\frac{z_2}{z_3} = \frac{3e^{-i\frac{\pi}{4}}}{\sqrt{2} e^{i\frac{2\pi}{3}}}$

$\frac{z_2}{z_3} = \frac{3}{\sqrt{2}} \times \frac{e^{-i\frac{\pi}{4}}}{e^{i\frac{2\pi}{3}}}$

$\frac{z_2}{z_3} = \frac{3}{\sqrt{2}} e^{-i\frac{\pi}{4} - i\frac{2\pi}{3}}$

$\frac{z_2}{z_3} = \frac{3}{\sqrt{2}} e^{i\left(-\frac{\pi \times 3}{4 \times 3} - \frac{2\pi \times 4}{3 \times 4}\right)}$

$\frac{z_2}{z_3} = \frac{3}{\sqrt{2}} e^{i\left(\frac{-3\pi - 8\pi}{12}\right)}$

$\boxed{\frac{z_2}{z_3} = \frac{3}{\sqrt{2}} e^{-i\frac{11\pi}{12}}}$

Exercice 85 page 255

1. $\frac{z_1}{z_2} = \frac{-1-i}{\frac{1}{2}+\frac{i\sqrt{3}}{2}}$

On multiplie en haut et en bas par le conjugué du dénominateur

$$\frac{z_1}{z_2} = \frac{(-1-i)\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}+i\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}-i\frac{\sqrt{3}}{2}\right)}$$

$$\frac{z_1}{z_2} = \frac{-\frac{1}{2}+i\frac{\sqrt{3}}{2}-\frac{1}{2}i-\frac{\sqrt{3}}{2}}{\boxed{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}}$$

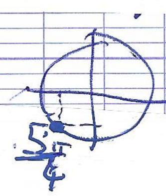
$\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4}+\frac{3}{4} = \boxed{1}$

$$\boxed{\frac{z_1}{z_2} = -\frac{1}{2}-\frac{\sqrt{3}}{2}+i\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)} \leftarrow \text{forme algébrique}$$

$z_1 = (-1)+(-1)i$

- $|z_1| = \sqrt{(-1)^2+(-1)^2} = \sqrt{1+1} = \sqrt{2}$

$\arg(z_1)$: $\begin{cases} \cos\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2} \\ \sin\theta = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2} \end{cases} \Rightarrow \theta = \frac{5\pi}{4}$

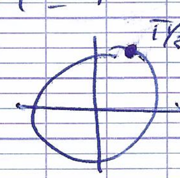


$$\boxed{\text{Donc } z_1 = \sqrt{2}e^{i\frac{5\pi}{4}}}$$

$z_2 = \frac{1}{2}+i\frac{\sqrt{3}}{2}$

- $|z_2| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{1}{4}+\frac{3}{4}} = \sqrt{1} = 1$

$\arg(z_2)$: $\begin{cases} \cos\theta = \frac{1}{2} \\ \sin\theta = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow \theta = \frac{\pi}{3}$



$$\boxed{\text{Donc } z_2 = 1e^{i\frac{\pi}{3}}}$$

forme expo → Ainsi $\frac{z_1}{z_2} = \frac{\sqrt{2}e^{i\frac{5\pi}{4}}}{1e^{i\frac{\pi}{3}}} = \sqrt{2}e^{i\frac{5\pi}{4}-i\frac{\pi}{3}} = \sqrt{2}e^{i\left(\frac{5\pi\times 3}{4\times 3}-\frac{\pi\times 4}{3\times 4}\right)} = \boxed{\sqrt{2}e^{i\frac{11\pi}{12}}}$

2. On sait que $\frac{z_1}{z_2} = \sqrt{2}e^{i\frac{11\pi}{12}} = \sqrt{2}\left(\cos\left(\frac{11\pi}{12}\right)+i\sin\left(\frac{11\pi}{12}\right)\right)$

$$\frac{z_1}{z_2} = \boxed{\sqrt{2}\cos\left(\frac{11\pi}{12}\right)}+i\boxed{\sqrt{2}\sin\left(\frac{11\pi}{12}\right)}$$

Or $\frac{z_1}{z_2} = \boxed{-\frac{1}{2}-\frac{\sqrt{3}}{2}}+i\boxed{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)}$

Ainsi $\boxed{\cos\left(\frac{11\pi}{12}\right) = \frac{-\frac{1}{2}-\frac{\sqrt{3}}{2}}{\sqrt{2}}}$ et $\boxed{\sin\left(\frac{11\pi}{12}\right) = \frac{\frac{\sqrt{3}}{2}-\frac{1}{2}}{\sqrt{2}}}$